NEEDS 2009 Workshop
Nonlinear Evolution Equations and Dynamical Systems
Isola Rossa, May 16–23, 2009.

Topologically Ordered States and their Hamiltonians

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February 26, 2009

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Continuous phase transitions between disordered and ordered phase states are accompanied by spontaneous symmetry breaking. The Landau theory of phase transitions, which describes this phenomenon, is based on the use of the local order parameters with non-zero value in the states with the decreased symmetry. However this theory cannot describe all continuous phase transitions between quantum-ordered states. This happens because in the systems with developed quantum fluctuations, phase transitions between disordered phases may take place at zero temperature without symmetry breaking. In this phenomenon, the initial and the final topologically ordered states are a special case of the order, which is characterized only by gapped excitations.

The study of topologically ordered states and quantum phase transitions between them is important for classifying various phase states in low-dimensional systems, where the role of quantum fluctuations is significant. In this case new types of ordering of strongly correlated spin degrees of freedom may be based on the employ of topological features of dynamics of excitations in low-dimensional systems; in particular, on the use of the effect of braiding of excitation world lines [1]. In spatially two-dimensional systems with internal degrees of freedom, braiding phenomena lead to non-Abelian statistics of excitations and to the corresponding form of the topological order. In the systems with non-Abelian quasiparticles, strongly correlated states form a certain part of low-energy Hilbert space. When the considered quasiparticles are spatially separated, a low-energy space turns to be degenerated, and its states are characterized by pure topological quantum numbers.

Determination of the maps between the Hamiltonians of exactly solvable quantum models [2, 3], study of correlation functions[4], as well as the classification of the topological order [5] and quantum phase transitions constitute incomplete list of problems in this field.

We consider a universal form for Hamiltonians of the systems, which are in the topologically ordered phase state. It is shown that in strongly correlated systems the Hamiltonian has a form of a sum of the projectors expressed through monoids of the generalized Temperly-Lieb algebra. In the case of twice linked excitation world lines it has a form of a two-dimensional Bloch matrix. In the limit of the infinite value of the linking degree, the system turns into the ordinary Heisenberg spin-1/2 model or into the biquadratic spin-1 one. Hamiltonians of the Fibbonacci anions [6] and their counterparts correspond to the intermediate values of the linking degree.

This work was supported in part by the E.I.N.S.T.E.IN grant (M.L., A.P.), RFBR grants (A.P., V.V.) Nos. 09-01-00268, 09-01-92426, and the program "Fundamental problems of nonlinear dynamics" of Presidium of the Russian Academy of Sciences.

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