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Analytic theory of self-similar mode-locking with rapidly varying, mean-zero dispersion

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Many optical systems are capable of exhibiting such self-similar behavior. Only recently has self-similar evolution of parabolic pulses been observed in mode-locked laser cavities operating with a nearly mean-zero dispersion map [1]. Here we present an explicit theoretical description of the *temporal* parabolic profiles observed in such lasers.

Pulse propagation in a laser cavity with rapidly varying dispersion can be described by the cubic-quintic Ginzburg-Landau (GL) equation [2]

$$iu_z+\frac{1}{2}d(z/\epsilon)u_{tt}+|u|^2u=i\delta u+i\beta|u|^2u+i\alpha u_{tt}-i\sigma|u|^4u, \qquad (1)$$

where z is the propagation distance, t is the time in a co-moving frame normalized by the average dispersion length, and u is the complex envelope of the electric field. The parameter $\epsilon \equiv P/T_0 \ll 1$ is the ratio of the laser cavity length to the dispersion length. A multi-scale transformation technique shows the leading order self-similarity to be governed by a nonlinear diffusion equation with a rapidly-varying, mean-zero diffusion coefficient. This equation has exact solutions of the form

$$|u|^2\approx\rho(x,t)\sim\frac{1}{12(\gamma+z_*)^{1/3}}\left[a_*^2-\left(\frac{(t-t_*)}{(\gamma+z_*)^{1/3}}\right)^2\right]_+ \qquad (2)$$

where $\gamma=\gamma(z)=2\int_0^z[\int_0^s d(q)dq]ds$ and $f_+=\max(f,0)$. The alternating sign of the diffusion coefficient, which is driven by the dispersion fluctuations, is critical to supporting the parabolic profiles which is, to leading-order, of the Barenblatt form. Further inclusion of dissipative terms reveals a reduced diffusion equation for the amplitude

$$\rho_z=\mu(z)(\rho^2)_{tt}-2\rho(\delta-\beta\rho+\sigma\rho^2). \qquad (3)$$

This reduced equation has parabolic solutions which act as stable attractors for a wide range of parameter space. This is the first analytic model proposing a mechanism for generating temporal parabolic pulses in the Ginzburg-Landau model.

References

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