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Matrix polynomials satisfying first order differential equations and three term recurrence relations

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We describe families of matrix valued polynomials satisfying simultaneously a first order differential equation and a three term recurrence relation (see [1]). Our goal is to address the classification of the matrix valued polynomials satisfying first order differential equations through the solutions of the so called *bispectral problem*.

The bispectral problem was first considered in the scalar case by J. J. Duistermaat and A. Grünbaum ([3]) when one differential operator has order two and the other differential operator has arbitrary order. In the case when the functions in question are matrix valued a first exploration of the power of this method is carried out in by A. Grünbaum and P. Iliev (see [4]). In [2] there was considered the bispectral situation for matrix polynomials when both operators, the difference and the differential one, have order one.

At the heart of this lies the need to solve some complicated nonlinear equations with matrix coefficients called *ad-conditions*. The solutions of these equations are studied under a variety of sufficient conditions on its coefficients.

References

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