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Perturbative techniques for integrable systems

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There are two main approaches to the perturbative study of integrable PDEs: 1) perturbations of linear PDEs and 2) perturbations of nonlinear integrable systems of hydrodynamic type. In the talk we will mainly consider the problems and results related to the second approach.

After explaining the main geometrical techniques such as the group of generalized Miura transformations, the idea of quasitriviality etc. we will concentrate on the study of phase transitions from regular to oscillatory behaviour of solutions to nonlinear Hamiltonian PDEs. Such phenomena were first observed for solutions to the Korteweg - de Vries (KdV) equation. The problem of such a critical behaviour in more general nonintegrable PDEs and systems of PDEs remains essentially unexplored. We propose simple arguments, partially supported by rigorous results as well as numerical evidences, that even in the general case a kind of local integrability holds at the point of phase transition. This gives a possibility to obtain an asymptotic description of the critical behaviour in terms of certain particular solutions of the Painlevé equations and their generalizations.

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