NEEDS 2009 Workshop
Nonlinear Evolution Equations and Dynamical Systems
Isola Rossa, May 16–23, 2009.

Nonlinear waves and wave scattering in layered elastic waveguides

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February 25, 2009

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1 Introduction

Layered (or laminated) structures are increasingly used in modern technologies, most notably in aerospace industry and microelectronics. Poor adhesion or delamination can lead to a catastrophic failure of the whole structure. Can nonlinear waves be used for nondestructive testing of such structures? Our current research is aimed at (a) derivation of *nonlinear long-wave models* for longitudinal waves in layered elastic waveguides with different types of bonding, (b) analytical and experimental studies of the *scattering of nonlinear waves* by the inhomogeneities.

2 Main results

I. Models

We propose a composite lattice model to describe nonlinear waves in a two-layered waveguide with adhesive bonding [1, 2]. Is it possible to derive some relatively simple systems of coupled nonlinear PDEs as *nonlinear asymptotic models* from a composite lattice, which has *all essential degrees of freedom* of a real layered elastic structure, and both *physical* and *geometrical* sources of nonlinearity? We answer this question by asymptotically deriving two new model systems of coupled Boussinesq-type equations [1]. Based on the shown similarity between continuum models for our basic lattice and a macroscopic waveguide, we conjecture that similar description exists within the framework of dynamic nonlinear elasticity. Our approach can be applied to other lattice models, e.g., used to study nonlinear waves or in molecular dynamics simulations.

II. Generalized Solitary Waves

We study nonlinear waves in a two-layered imperfectly bonded structure using one of the derived systems of coupled Boussinesq-type equations [2]

$$f_{tt} - f_{xx} = \frac{1}{2}(f^2)_{xx} + f_{ttxx} - \delta(f - g),$$
$$g_{tt} - c^2 g_{xx} = \frac{1}{2}\alpha(g^2)_{xx} + \beta g_{ttxx} + \gamma(f - g),$$

and find two nontrivial conservation laws, which are later used to control the accuracy of our numerical simulations. We show that the *classical* solitary waves, which exist in the symmetric case, are structurally unstable. They are replaced with *generalized* solitary waves, having co-propagating oscillatory *long-wave* tails. We discus a possible link with recent experimental results in [3].

III. Scattering

We pose a mathematical problem, describing scattering of long nonlinear longitudinal waves in a layered elastic waveguide with poor adhesion or delamination:

$$\begin{aligned}
&u^-_{tt} - u^-_{xx} = 2\varepsilon[-6u^-_x u^-_{xx} + u^-_{xxxx}] \quad \text{for} \quad x < 0,\\
&u^+_{tt} - c^2 u^+_{xx} = 2\varepsilon[-6\beta u^+_x u^+_{xx} + \gamma u^+_{xxxx}] \quad \text{for} \quad x > 0,\\
&u^-|_{x=0} = u^+|_{x=0},\\
&u^-_x + 2\varepsilon[-3(u^-_x)^2 + u^-_{xxx}]|_{x=0} = c^2 u^+_x + 2\varepsilon[-3\beta(u^+_x)^2 + \gamma u^+_{xxx}]|_{x=0}.
\end{aligned}$$

To solve this problem we develope an approach which uses a combination of matched asymptotic multiple-scales expansions, integrability theory of the KdV equation by the Inverse Scattering Transform and some natural radiation conditions [4]. We show, both in theory and experiments [4] - [7], that splitting of a layered structure leads to the generation of a train of secondary solitary waves from a single incident soliton, and thus, *can be potentially used for nondestructive testing of layered structures.*

3 Acknowledgments

This work was done in collaboration with G.V. Dreiden, A.M. Samsonov and I.S. Semenova from the Ioffe Institute of the Russian Academy of Sciences, and A.S. Zakharov from Loughborough University, and was partially supported by the UK EPSRC under the grant EP/D035570/1.

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