NEEDS 2009 Workshop
Nonlinear Evolution Equations and Dynamical Systems
Isola Rossa, May 16–23, 2009.

Autophasing of Solitons

S.V. Batalov[a], L.A. Kalyakin[b], E.M. Maslov[c]
A.G. Shagalov[a]

March 2, 2009

a. Institute of Metal Physics of the Russian Academy of Sciences, Ekaterinburg, Russia.

b. Institute of Mathematics of the Russian Academy of Sciences, Ufa Scientific Center, Ufa, Russia.

c. IZMIRAN, Troitsk, Moscow Region, Russia.

We investigate the effect of small external quasiperiodic perturbations with slowly varying frequency on localized time-periodic solutions of the Nonlinear Schrodinger (NS) equation,

$$iu_t + \frac{1}{2}u_{xx} + |u|^2 u = \varepsilon e^{ikx+i\psi(t)}, \tag{1}$$

and the Sine-Gordon (SG) equation,

$$u_{tt} - u_{xx} + \sin u = \varepsilon \cos \psi(t). \tag{2}$$

Here $0 < \varepsilon \ll 1$, $\psi_t = \Omega(t)$, $\Omega_t = O(\alpha)$, $|\alpha| \ll 1$, k is a fixed wavenumber. In such systems, under certain initial conditions and with certain parameters of the perturbations, the resonant phaselocking phenomenon can occur. In this case the frequency of internal oscillations of a localized solution follows the frequency of an external perturbation, while the amplitude of the solution grows or decreases depending on the sign of Ω_t. To describe this effect in the first order in ε we seek solutions of eqs. (1) and (2) in the form

$$u(x,t) = \varphi(x,t) + \chi(x,t), \tag{3}$$

where φ is a perturbed soliton,

$$\varphi_s(x,t) = \frac{A}{\mathrm{ch}\,[A\,(x-\xi)]} e^{i[V(x-\xi)+\theta]}, \tag{4}$$

or a perturbed breather,

$$\varphi_b(x,t) = -4 \arctan\left[\tan A \frac{\cos\theta}{\cosh(x \sin A)}\right], \tag{5}$$

correspondingly, $\chi(x,t) \sim \varepsilon$ is a small wave induced by the nonlocalized perturbations standing in (1), (2). The functions $A(t), V(t), \xi(t), \theta(t)$ are to be found. Using the perturbation theory, based on the inverse scattering transform [1, 2], we obtain the systems of equations for these functions in both cases. The systems are similar in Hamiltonian structure and reduce to the form

$$A_t = \varepsilon G(A, \Omega) \sin \delta, \tag{6}$$

$$\delta_t = \Delta\omega + \varepsilon \frac{\partial G(A, \Omega)}{\partial A} \cos \delta, \tag{7}$$

where δ is a phase difference, $\Delta\omega(A, \Omega)$ is a frequency detuning, $\Omega(t)$ is a slowly varying frequency of an external perturbation, and the function $G(A, \Omega)$ is specified by the NS and SG cases. We investigate the system (6), (7) in the so-called nonlinear pendulum approximation and derive the necessary and sufficient conditions for the phaselocking [3, 4]. The results are in a good agreement with the ones obtained by the direct numerical integration of the perturbed NS and SG equations.

References

[1] Karpman, V.I. and Maslov, E.M.: Perturbation Theory for Solitons. *Sov. Phys.-JETP.* **46** (1977) 281.

[2] Karpman, V.I., Maslov, E.M. and Solov'ev, V.V.: Dynamics of Bions in Long Josephson Junctions. *Sov. Phys.-JETP.* **57** (1983) 167–173.

[3] Maslov, E.M., Kalyakin, L.A. and Shagalov, A.G.: Breather Resonant Phase Locking by an External Perturbation. *Theor. Math. Phys.* **152** (2007) 1173–1182.

[4] Batalov, S.V., Maslov, E.M. and Shagalov, A.G.: Autophasing of Solitons. *JETP.* **135** (2009) in press.