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Integrability of Second-order Ordinary Differential Equations through $\lambda-$Symmetries

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1 Abstract

Recently ([1]) we have obtained a novel derivation of first integrals and integrating factors for ordinary differential equations (ODEs) based on the $\lambda-$symmetries of the equation ([2]). As a consequence, once a $\lambda-$symmetry of the equation is known, an algorithm to calculate integrating factors and first integrals is derived. This algorithm represents a significant simplification of the calculations derived by other methods ([3],[4],[5],[6]).

In this paper we explore the method of finding general solutions of second-order equations through its $\lambda-$symmetries. If two non equivalent $\lambda-$symmetries of a second-order ODE are known, we can find find two independent first integrals and hence, the general solution of the given ODE. This method includes the well-known method of Lie point symmetries ([7],[8]), but can also be applied to ODEs with trivial algebra of Lie point symmetries.

We illustrate the theory with several examples and explore their underlying solutions. Particular nonlinear ODEs representing physically important oscillators systems are included in our study. Hopefully this new procedure may provide further understanding of these models.

References

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