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Non-Autonomous Dynamics of a Nonlinear Spin-Torque Nano-Oscillator

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1 Spin-Torque Nano-oscillators (STNO)

The discovery of the spin-transfer torque effect by J. Slonczewski [1] and L. Berger [2] opened a possibility for a new method of generation of microwave oscillations that does not involve any semiconductor materials or devices. It turned out, that electric direct current passing through a magnetized magnetic layered structure becomes spin-polarized and, if the current density is sufficiently high, this spin-polarized current can transfer enough spin angular momentum between the magnetic layers to de-stabilize the static equilibrium orientation of magnetization in the thinner ("free") magnetic layer of the multi-layered structure and to start self-sustained oscillations of magnetization. Thus, using the spin-torque effect it is possible to create a novel type of nano-sized microwave oscillators – spin-torque nano-oscillators (STNO) [3].

Understanding of non-autonomous dynamics of STNOs subjected to external perturbations, such as thermal noise or periodic signals, is of a critical importance for the practical applications of STNO. In particular, thermal noise determines the STNO generation linewidth, while the action of periodic signals can lead to the phase-locking of STNO to an external frequency. The Landau-Lifshitz-Gilbert-Slonczewski equation [1], which describes the STNO dynamics, is nonlinear and can not be solved analytically in a general case. Therefore, the STNO dynamics is, usually, studied numerically. However, numerical simulations of non-autonomous STNO dynamics, especially in the case of STNOchastic external signals (like thermal noise), are rather difficult and time-consuming. Also, for every new external perturbation these calculations must be repeated from the very beginning.

2 Forced Dynamics of STNO

In this work, we propose a perturbative analytic approach to the description of non-autonomous STNO dynamics which is based on using the autonomous (unperturbed) STNO dynamics, obtained either analytically or numerically, as a zero approximation. The role of a small parameter is played by the ratio of the energy of the STNO interaction with an external perturbation to the energy of the autonomous (unperturbed) STNO motion. The proposed approach is valid for any STNO geometry and any amplitude of the autonomous STNO motion. This approach is, also, valid for an arbitrary (but sufficiently small) external perturbation and, in particular, for the perturbation in the form of periodic signals coming from other STNOs forming a large STNO array.

In the developed approach, we use the fact that both non-conservative (Gilbert's and Slonczewski's) terms in the Landau-Lifshitz-Gilbert-Slonczewski equation [1] are small in comparison to the main (precessional) term, so the unperturbed STNO motion is quasi-Hamiltonian, and the competing non-conservative terms simply stabilize a particular STNO trajectory that is close to a Hamiltonian one. Thus, in the theory it is possible to introduce canonical variables for the STNO system: action J, which is a mathematical equivalent of the oscillation power, and the phase Ψ, which is canonically conjugated with J. In these variables the non-autonomous dynamics of a perturbed STNO is described by the following simple system of equations:

$$\frac{dJ}{dt} + \Gamma(J)(J - J_0) = -(\mathbf{K}_J(J) \cdot \mathbf{h}(t))\,, \tag{1}$$

$$\frac{d\Psi}{dt} - \omega_0(J) = (\mathbf{K}_\Psi(J) \cdot \mathbf{h}(t))\,, \tag{2}$$

were J_0 is the power of the autonomous (unperturbed) STNO motion, $\Gamma(J)$ is the decay rate of power perturbations in STNO, $\omega_0(J)$ is the STNO oscillation frequency dependent on the STNO oscillation power J, and $\mathbf{K}_J(J)$, $\mathbf{K}_\Psi(J)$ are the vector functions describing the influence of the external perturbation $\mathbf{h}(t)$ on the power J and phase Ψ of the STNO. Note, that the coefficients J_0, $\Gamma(J)$ and functions $\omega_0(J)$, $\mathbf{K}_J(J)$, $\mathbf{K}_\Psi(J)$ entering Eqs. (1) and (2) are determined from the solution of the autonomous dynamics problem of an unperturbed STNO.

To illustrate our approach we solved a problem of phase-locking of an STNO having a non-trivial geometry with two coupled (by both spin-torque and dipole-dipole interaction) ferromagnetic layers to an external periodic signal $\mathbf{h}(t)$. In the studied STNO geometry the spin-polarized current excites magnetization oscillations in both coupled magnetic layers, each of which is considered in the macrospin approximation. The spectrum of oscillations in this STNO geometry consists of two branches with different frequencies. Using Eqs. (1) and (2) we calculated the phase-locking effect for the low-frequency STNO branch in the case when the precession angles in both layers were not small.

The results of the developed analytic theory are in good agreement with the results of direct numerical simulations performed using the coupled Landau-Lifshitz-Gilbert-Slonczewski equations [1].

References

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