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Spatial dispersive shock waves in nonlinear optics

Stefano Trillo[a] Claudio Conti[b] Andrea Fratalocchi [c]

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a. Dept. of Engineering, University of Ferrara, Via Saragat 1, 44100 Ferrara, Italy.

b. Research Center SOFT INFM-CNR, Universita' di Roma La Sapienza, P. A. Moro 2, 00185 Rome, Italy.

c. Dipartimento di Fisica, Universita' di Roma La Sapienza, P. A. Moro 2, 00185 Rome, Italy.

1 General remarks

Shock waves, characteristic of hyperbolic PDEs, can be regularized by weak dispersive effects, featuring the onset of fast oscillations in an expanding region usually known as dispersive shock fan. We report on the experimental observation and relative theoretical understanding of a shock fan produced by a laser beam under the action of weak diffraction (playing the role of dispersion) and strong defocusing, the latter being due to a nonlinear response of thermal origin. At high laser powers, starting from a smooth transverse distribution, a gradient catastrophe along the beam profile occurs at a finite distance. Specifically we address the case of a dark input along which a phase jump is nested, contrasting this case with that of a bright input. The essential dynamics is shown in Fig. 1. While moderate nonlinearities [frame (b)] compensate for the diffractive spreading [frame (a)] and lead basically to 1-soliton formation, in the strongly nonlinear regime [frames (c-d)] the beam forms a focus (cusp or catastrophe) point beyond which it opens up into a shock fan whose transverse expanding oscillations exhibit features of dark narrow filaments, which propagate with characteristic transverse velocities and grow in number with the nonlinearity.

2 Theory

The analysis of the phenomenon is based on the following weakly dispersive nonlocal nonlinear Schrödinger equation,

$$i\varepsilon \frac{\partial \psi}{\partial z} + \frac{\varepsilon^2}{2} \frac{\partial^2 \psi}{\partial x^2} - n\,\psi = 0, \tag{1}$$

$$-\sigma^2 \frac{\partial^2 n}{\partial x^2} + n = |\psi|^2, \tag{2}$$

where ψ and n are the optical field and refractive index change, respectively, $\varepsilon \ll 1$, and σ is a free parameter which measures the degree of nonlocality of

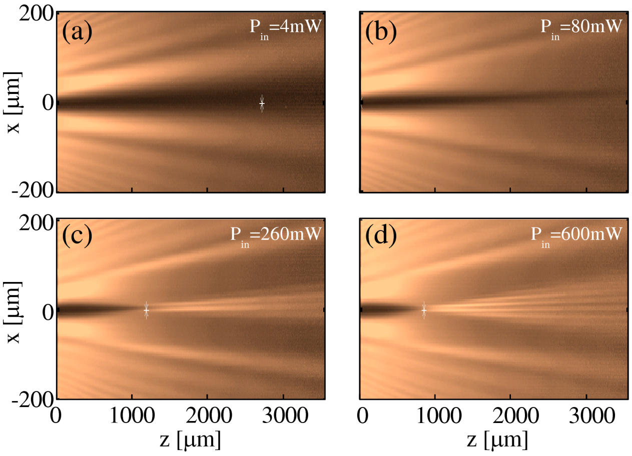

Figure 1: Evolution along distance z (in a cell filled with dye-doped methanol) of a transverse dark notch (at $x = 0$) as nonlinearity grows: (a) linear regime; (b) moderately nonlinear regime; (c-d) formation of a gradient catastrophe (focus dark point) and shock fan. See Ref. [2] for details.

the nonlinear response. The dynamics up to the shock point, along with an estimate of its location, can be described by a WKB reduction to the hydrodynamical limit of Eqs. (1). Beyond the shock point, different approaches help to understanding the phenomenon. In the local integrable limit $\sigma = 0$, the inverse scattering transform can be applied. In this case, the dispersive shock can be interpreted as a radiation-less phenomenon in terms of $(2N - 1)$-soliitons, N being the input to fundamental soliton amplitude ratio. A counterpart of the phenomenon is the occurence of a first-order transition as can be shown by applying methods of the statistical mechanics of chaos in a phase-space associated with eigenvalues [1]. Although the scenario remains basically unchanged in the diffusive (nonlocal, $\sigma \neq 0$) case [2], a proper account of the details enlighten by the numerics, is likely to require the application of Whitham modulation approach. The nonlocality appears to play a crucial role for observing such wave-breaking scenario in two transverse dimensions (input stripe), since it suppresses transverse (snake) instabilities.

Finally we will also discuss perspectives and future direction of this work.

References

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[2] A. Fratalocchi, C. Conti, G. Ruocco, and S. Trillo, *Free-energy transition in a gas of noninteracting nonlinear wave particles*, Phys. Rev. Lett. **101**, 044101 (2008).