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Symbolic representation and classification of integrable systems

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In this talk we would like to give a brief account of recent development of the symmetry approach [1, 2]. The progress has been achieved mainly due to a symbolic representation of the ring of differential polynomials which enable us to use powerful results from algebraic geometry and number theory. Symbolic representation (an abbreviated form of the Fourier transformation) has been originally applied to the theory of integrable equations by Gel'fand and Dikii [3]. Symmetry approach in symbolic representation has been formulated and developed to tackle the problem of the global classification of integrable evolutionary equations in [4, 5, 6]. In symbolic representation the existence of infinite hierarchy of symmetries is linked with factorisation properties of an infinite sequence of multi-variable polynomials. Symbolic representation is suitable for studying integrability of noncommutative [7], non-evolutionary [8, 9, 10], non-local (integro–differential) [11], multi–component [12, 13, 14] and multi–dimensional equations [15]. It provides a powerful tool for testing integrability of a given system. It enables us to obtain the intrinsic structure of the symmetry hierarchy and global classification results of integrable systems.

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