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Quantum monodromy and pattern formation

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1 Hamiltonian monodromy

Integrable Hamiltonian models are widely used in classical and quantum physics as qualitatively reliable starting point to understand dynamical behavior of the system. Hamiltonian monodromy is known to be the first obstruction to the existence of global action coordinates in integrable systems [1, 2]. It is important that monodromy remains relevant as well to nearly integrable Hamiltonian systems [3]. Serious interest in Hamiltonian monodromy studies was stimulated recently by numerous manifestations of monodromy in many fundamental quantum physical atomic and molecular problems as hydrogen atom [4], angular momentum coupling [5], rovibrational structure of simple molecules [6], and many others. The most straightforward observation of Hamiltonian monodromy in quantum problems is through the analysis of the patterns formed by a common spectrum of mutually commuting quantum observables where the monodromy shows itself as a specific defect of the almost regular lattice [7].

2 Possible generalizations

Along with looking for new physical examples of manifestations of Hamiltonian monodromy, different kinds of generalizations were suggested last years. One way of generalizations was based essentially on the construction of new generic singularities of toric fibrations which result in new defects like fractional monodromy [8] or bidromy [9] or in defects without monodromy [10]. Alternatively, instead of "static" manifestation of monodromy its implication into time dependent evolution of dynamical systems was studied [11].

After a brief introduction of Hamiltonian monodromy and its manifestation in simple static and time-dependent systems I want to discuss a new possible example of relation between monodromy and time-evolution of a complex system which is rather far from all recently studied monodromy applications. It is very old and phenomenologically well known phenomenon of spiral phyllotaxis [12] which nevertheless was never previously discussed from the point of view of monodromy of the defects associated with natural arrangements of leaves and seeds in a grand variety of plants. Analysis of the regular part of pattern formed, for example, by seeds in a sunflower from the point of view of "static monodromy" shows that an elementary cell after going around a closed path surrounding flower center return back to its original position after making a

2π rotation around itself. This means that the corresponding defect is highly nontrivial in spite of the fact that its monodromy is just an identity. It is interpreted as a cumulative effect of 12 elementary monodromy defects of the same sign and this sign which is well defined for 2D-Hamiltonian systems is exactly the sign allowed for the generic "elementary monodromy defect" by Hamiltonian dynamics. Construction of an evolutionary equation [13] leading to stationary pattern with such specific defect can be formulated as a next challenge problem related to monodromy related applications.

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